REFERENCES

1. T. Zaleski and A. B. Jarzebski, Remarks on some properties of the equation of heat transfer in multichannel exchangers, *Int. J. Heat Mass Transfer* **16**, 1527–1530 (1973).
2. A. Settari, Remarks about "General solution of the equations of multichannel heat exchangers", *Int. J. Heat Mass Transfer* **15**, 555–557 (1972).
3. J. C. Chato, R. J. Laverman and J. M. Shah, Analysis of parallel flow, multistream heat exchangers, *Int. J. Heat Mass Transfer* **14**, 1691–1703 (1971).

Int. J. Heat Mass Transfer. Vol. 17. pp. 1118–1119. Pergamon Press 1974. Printed in Great Britain

ANALYSIS OF LOWE'S MEASUREMENTS OF EFFECTS OF VIBRATION ON HEAT TRANSFER

P. D. RICHARDSON* and K. TANISHITA†

Center for Fluid Dynamics, Division of Engineering, Brown University, Providence, R.I., U.S.A.

(*Received* 6 *November* 1973)

NOMENCLATURE

a,	amplitude of oscillation;
d,	cylinder diameter;
g,	acceleration due to gravity;
H,	heat-transfer factor (defined in text);
h,	heat-transfer coefficient;
k,	thermal conductivity;
R,	cylinder radius;
ΔT,	difference in temperature between cylinder surface and distant air;
β,	thermal coefficient of expansion of air;
ε,	$4Re_s/Gr^{1/2}$;
λ,	ratio of thicknesses, natural convection boundary layer to harmonic oscillation boundary layers;
ν,	kinematic viscosity;
ω,	circular frequency of imposed harmonic oscillation;
Gr,	Grashof number, $g\beta\Delta TR^3/\nu^2$;
Nu,	Nusselt number, hd/k;
Re_s,	streaming Reynolds number, $a^2\omega/\nu$.

RECENT analysis of the effects of vibrations on heat transfer occurring otherwise by pure natural convection have shown a significant role should be played by a boundary-layer thickness parameter λ [1]. Experimental data obtained by Lowe [2] happen to fall in a range of λ where effects of λ should be strong, and allow some comparisons which are described here.

Effects of vibration and sound fields on heat and mass transfer have often been measured for a circular cylinder with the oscillations transverse to its axis. There are several different classes of flow situations which can dominate the transfer process, and no single correlation can reasonably be used to fit all the data [3]. Even in the absence of natural convection effects, data may fall into at least three distinct characteristic solutions in one of which convection is dominated by outer streaming at large streaming Reynolds numbers [4]. Analysis of combined natural convection and horizontal or vertical oscillations at a heated horizontal cylinder [1] for large Grashof and large streaming Reynolds numbers, predicts local changes in boundary-layer thickness and heat transfer; these changes correspond to the directions of local changes observed in experiments at large Grashof and small streaming Reynolds numbers [5].

The analysis [1] draws attention to the additional characteristic parameter $\lambda = R/Gr^{1/4}(2\nu/\omega)^{1/2}$, the ratio of the natural convection boundary-layer thickness to the oscillating boundary-layer thickness. At any finite value of $\varepsilon = 4\,Re_s/Gr^{1/2} = 4(a^2\omega/\nu)/Gr^{1/2}$, the change in heat-transfer increases with λ, rapidly at small values of $\lambda(<10$, say), and approaches a finite limit as $\lambda \to \infty$.

Experimental data at large values of streaming Reynolds number $Re_s = a^2\omega/\nu$ are most easily obtained when ω is small because a can be quite large then. When ω is small, however, the oscillation boundary-layer thickness $(2\nu/\omega)^{1/2}$ is relatively large and values of λ may typically fall in the range 2 to 10, which complicates correlation of data because of the relatively strong effect of λ in this range. At present there does not seem to be local data available at large Re_s to compare with the predictions of the analysis that horizontal oscillations increase the heat-transfer rate at the bottom of the cylinder, and that vertical oscillations decrease the heat transfer, as ε is increased from zero to moderate values. (As ε becomes large, one must expect effects of natural convection to become unimportant.) However, the unpublished thesis of Lowe [2] contains data for overall heat transfer at moderate Grashof numbers (about 3×10^3) and with streaming Reynolds numbers up to 800 ("large"), and it is worthwhile examining these data in the light of the analyses.

The following points can be established about Lowe's data:

(1) The experimental results merge with the two pertinent asymptotes. Figure 1, in which $(Nu/Gr^{1/4})\{1+0{\cdot}94(a/d)\}$ is plotted as a function of ε, shows how the data approach the asymptotic cases of $\varepsilon \to 0$ (pure natural convection) and $\varepsilon \to \infty$ (acoustic streaming dominant) [4]. The value of $Nu/Gr^{1/4}$ expected as $\varepsilon \to 0$ is somewhat larger than that for large Gr (i.e. for the boundary-layer solution as $Gr \to \infty$) because of boundary-layer curvature effects [6].

(2) The rise of *overall* heat transfer as ε increases from zero is slower than the change of local heat transfer predicted by analysis [1] for the same range of λ. A similar observation has been found in other experiments with much higher values of λ [5, 7], where it was found that there are simultaneous local changes of opposing sign and similar magnitude at different locations around the cylinder.

---

*Professor of Engineering.

†Research Assistant.

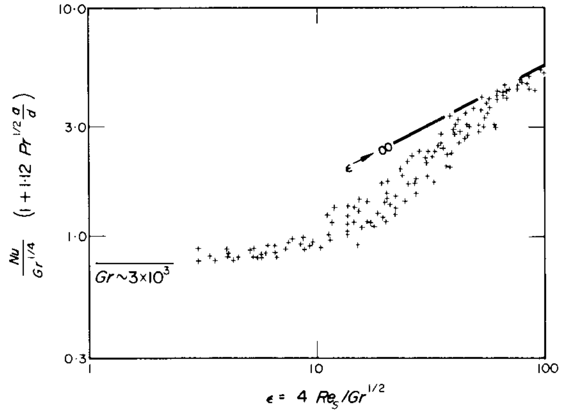

FIG. 1. Lowe's experimental results plotted as overall heat transfer as a function of a parameter which characterizes the relative strength of streaming motion to natural convection, ε. The parameter in braces on the vertical axis is a correction factor which allows for the effect of pure conduction in the inner streaming region, an effect omitted in the analysis for $\varepsilon \to \infty$. Lines are drawn for asymptotic expectations: as $\varepsilon \to 0$, the line for natural convection at $Gr \sim 3 \times 10^3$ is shown, and for $\varepsilon \to \infty$ the line for convection by acoustic streaming at large streaming Reynolds numbers (4) is shown.

(3) The measured change in overall heat transfer at moderate values of ε is found to decrease as the value of λ increases; the change in local heat transfer found by analysis at small values of ε increases as λ increases. To investigate the effect of λ in Lowe's data, a statistical analysis was applied. Results for $10 < \varepsilon < 30$ were selected, because data in this range show heat transfer is clearly in excess of natural convection without having reached the asymptotic limit for streaming alone. The quantity

$$H = \{(hd/k)/\varepsilon^{1/2}(g\beta\Delta T R^3/\nu^2)^{1/4}\}\{1+0{\cdot}94(a/d)\}$$

was calculated for each data point, and data points were grouped in sets depending on the value of λ for the point; for set 1, $2{\cdot}0 < \lambda < 2{\cdot}5$; for set 2, $2{\cdot}5 < \lambda < 3{\cdot}0$; for set 3, $3{\cdot}0 < \lambda < 3{\cdot}5$; and for set 4, $3{\cdot}5 < \lambda < 4{\cdot}0$. For both horizontal and vertical vibrations, the mean values of H for each set decreased with increasing level of λ for the set. "Student's" t-test [8] was applied to estimate the confidence level that these differences represent different populations of data (i.e. that the differences are systematic). Table 1 lists confidence levels for comparison of various sets. It can be seen that the decrease of heat transfer in Lowe's data as the value of λ is increased is shown to a high level of confidence.

Table 1

Sets compared	1–2	1–3	1–4
Confidence level % (horizontal oscillations)	98·6	99·2	99·1
Confidence level % (vertical oscillations)	93·0	98·6	97·5

Data of Fand *et al.* for horizontal [9] and for vertical [10] vibrations suggest similar trends, but their range of λ and number of points are too small to apply statistical analysis usefully.

(4) The measured change in *overall* heat transfer at moderate values of ε involves increase above pure natural convection for both horizontal and vertical vibrations; in matched ranges of λ, the overall heat transfer (as indicated by values of H, defined in (3) above) is slightly greater with horizontal vibrations than with vertical. Under a t-test, the confidence level of this comparison ranges from about 70 to 90 per cent.

In summary, Lowe's data fit well within the spectrum of experimental results for heated cylinders in the presence of vibrations and sound and agree with asymptotic analysis for natural convection and acoustic streaming. In particular, the data demonstrate the significance of the boundary-layer parameter λ at intermediate values of ε. However, the data do not permit a quantitative comparison with local analysis available in the latter range because the simultaneous local changes of opposing sign and similar magnitude at different azimuthal locations serve to mask details of local effects. Thus even the different directions of change of local heat transfer at the bottom stagnation region found in analysis and in experiments [7] for horizontal and vertical vibrations are seen only as a slightly smaller increase in overall heat transfer for vertical contra horizontal oscillations in Lowe's overall heat-transfer data. These observations serve to emphasize that it is essential either to expand the analysis to account for local changes all around a cylinder or to make local heat-transfer measurements for comparison with the existing analysis which provides predictions for the bottom region of a cylinder.

Acknowledgement The authors are grateful to Mr. Lowe for permission to quote the results presented in his thesis.

REFERENCES

1. G. de Vahl Davis and P. D. Richardson, Natural convection in a sound field giving large streaming Reynolds numbers, *Int. J. Heat Mass Transfer* **16**, 1245–1265 (1973).
2. H. C. Lowe, Heat transfer from a vibrating horizontal tube to air, M.Sc.(Eng.)thesis, University of London (1965).
3. P. D. Richardson, Effects of sound and vibrations on heat transfer, *Appl. Mech. Rev.* **20**, 201 (1967).
4. B. J. Davidson, Heat transfer from a vibrating circular cylinder, *Int. J. Heat Mass Transfer* **16**, 1703 (1973).
5. P. D. Richardson, Local details of the influence of a vertical sound field on heat transfer from a circular cylinder, *Proc. 3rd Int. Heat Transfer Conf.* **3**, 71 (1966).
6. J. A. Peterka and P. D. Richardson, Natural convection from a horizontal cylinder at moderate Grashof numbers, *Int. J. Heat Mass Transfer* **12**, 749 (1969).
7. P. D. Richardson, Local effects of horizontal and vertical sound fields on natural convection from a horizontal cylinder, *J. Sound Vib.* **10**, 31 (1969).
8. E. B. Wilson, *An Introduction to Scientific Research*, Chapters 8–9. McGraw-Hill, New York (1952).
9. R. M. Fand and E. M. Peebles, A comparison of the influence of mechanical and acoustical vibrations on free convection from a horizontal cylinder, *J. Heat Transfer* **84**, 268 (1962).
10. R. M. Fand and J. Kaye, The influence of vertical vibrations on heat transfer by free convection from a horizontal cylinder, *Int. Dev. Heat Transfer* 490 (1961).